An Electromechanical Model for MEMS Switches

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***Abstract* — This paper presents a new dynamic and parametric model of a capacitive micro-electromechanical switch electro-statically actuated. It is built of elementary electrical equivalent-circuit blocks constructed of voltage-controlled current sources. The model describes the non-linear mechanical behavior of the membrane movement, which allows the computation of the bridge capacitance. It can be inserted as a model to perform simulations at microwave frequencies. All the switch parameters such as dimensions and materials properties are adjustable.**

I. Introduction

The development of micro-electromechanical systems (MEMS) switch and their new applications in RF requires the development of equivalent models to be included as single elements in circuit simulators. Some models have been recently developed [1], [2] but they do not describe the mechanical behavior of the membrane and the behavior of the component at RF at the same time. Another way is to study the complete structures by coupled time domain mechanical – electromagnetic methods, but they require large computational time.

The non-linear behavior of a MEMS beam has been studied in the past [3] using non-linear voltage-controlled current sources, to model the mechanical behavior of mechanical resonators with commercial microwave circuit simulation software.

In this paper, a model using mechanical/electrical equivalencies to describe the complete behavior of a RF micromechanical switch is presented. The model computes the switch capacitance as a function of the different parameters (such as membrane length, width, thickness, initial height…) and as a function of a low frequency voltage applied between the membrane and the bottom electrode. This dynamic and parametric model allows both nonlinear mechanical simulation and S parameters simulation in commercial simulators package.

To validate the study, the model has been included in a distributed MEMS transmission line BPSK modulator circuit, designed by Barker and Rebeiz [6].

II. Electromechanical Model

The membrane can be considered as a mechanical system on which several forces are applied (Fig. 1) [4].

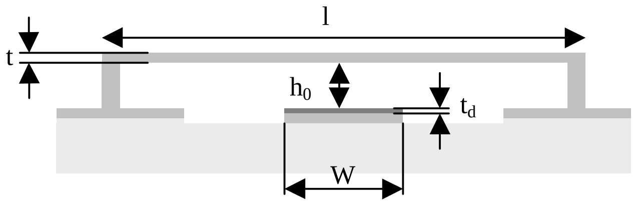

Fig. 1. Dimension of the modeled switch.

The motion's equation of the membrane is:

$$m\frac{d^2z}{dt^2} = -kz - \xi\frac{dz}{dt} + F_{el} \tag{1}$$

where m (kg) is the mass of the membrane, z (m) is the position of the membrane, k ($N.m^{-1}$) is the spring constant, ξ ($N.m^{-1}.s$) is the damping coefficient and F_{el} (N) is the non linear electrostatic force between the electrodes of the capacitance.

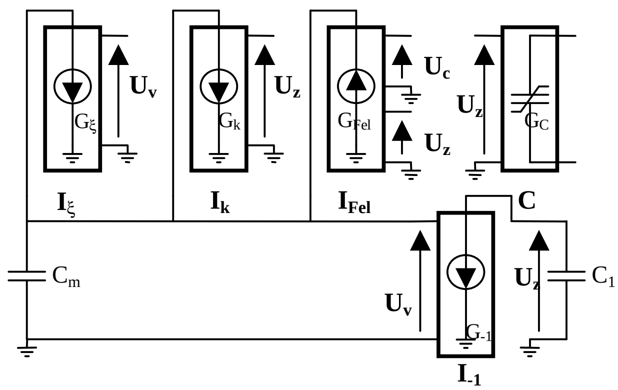

Fig. 2. Equivalent mechanical model of capacitive switch using voltage-controlled sources. The component values are given Table II.

The model is presented Fig. 2. The model uses voltage-controlled current sources to model the equation (1). Each current source represents one of the mechanical forces applied on the membrane. The use of voltage-controlled sources allows to model the different force non-linearity. The model takes three different forces into account: the spring force kz, the damping force ξdz/dt and the electrostatic force F_{el}.

The expressions of the spring constant k and the damping constant ξ are depending on the switch dimensions and of the material properties. The electrostatic force depends not only on the dimensions and on the material properties but also on the position of the membrane and on the voltage U_c applied between the membrane and the bottom electrode [4] and [5].

TABLE I
EQUATIONS

Mechanical	Eq. Model
Spring constant	$G_k = G_{k'} + G_{k''}$
Stretch	$G_{k'} = \frac{32Ewt^3}{l^3} \frac{1}{2-\left(2-\frac{W}{l}\right)\left(\frac{W}{l}\right)^2}$
Axial stress	$G_{k''} = \frac{8\sigma(1-\nu)wt}{l} \frac{1}{2-\frac{W}{l}}$
Damping coefficient	$G_\xi = \frac{\mu W w^3}{h_0^3}$
Electrostatic force	$G_{F_{el}} = \frac{Ww\varepsilon_0 U_c^2}{2(h_0-z)^2}$
Switch capacitance	$G_C = \frac{\varepsilon_0 wW}{h_0 - z + \frac{t_d}{\varepsilon_r}}$
	$G_{-1} = -1$
Beam mass	$C_m = lwt\rho$
	$C_1 = 1$
Pull down voltage	$V_p = \sqrt{\frac{8k}{27\varepsilon_0 Ww} h_0^3}$

In the model, the voltage U_v, which represents the speed of the membrane, is applied at the ports of the capacitance C_m (m F, where m is the beam mass in kg). This voltage controls the current source I_{-1} (conductance G_{-1}=-1 S), which delivers a current equal to $-U_v$. This current passes through the capacitance C_1 (1 F). The voltage U_z, which represents the displacement z (µm) of the membrane, is connected to the capacitance C_1. The voltage U_z is related to the current delivered by the source I_{-1} by the equation:

$$I_{-1} = C_1 \frac{dU_z}{dt} \quad (2)$$

If I_{-1} and C_1 are replaced by their values, the equation obtained is:

$$U_v = \frac{dU_z}{dt} \quad (3)$$

The voltage U_v controls the source I_ξ (conductance G_ξ, where G_ξ is the value of the damping coefficient ξ), which delivers a current equal to $G_\xi U_v$. This current represents the damping force. U_z controls the current source I_k (conductance G_k, where G_k is the value of the spring constant k), which delivers a current equal to $G_k U_z$. The current I_k represents the spring force. The electrostatic force is represented by the current source I_{Fel}. This source is a non-linear current source, controlled by the two voltages U_z and U_c. The expression of the source conductance G_{Fel}, which represents the electrostatic force is given in Table I. This conductance represents the fact that the electrostatic force depends on the displacement z (represented by U_z) and the voltage U_c applied between the membrane and the bottom electrode. The modeling of the electrostatic force requires a current source I_{Fel} controlled by two independent voltages. To create a such source the circuit software must allow to create current sources controlled by several independent voltages. For instance, it is achievable with a SDD in the circuit software Agilent-ADS.

The current, which passes through the capacitance C_m is equal to the sum of the currents delivered by the sources I_ξ, I_k, I_{Fel} and it is too equal to m*dU_v/dt. So, the equation obtained is:

$$m\frac{dU_v}{dt} = -I_k - I_\xi + I_{F_{el}} \quad (4)$$

which is equivalent to the equation (1).

As the mechanical equation is verified by the model, the displacement z of the membrane can be known (thank U_z) in function of the voltage U_c applied between the membrane and the bottom electrode. The capacitance of the switch can then be approximated using the expression given in Table I and full wave simulators can also be used for this purpose. In the model, the capacitance of the switch is represented by a charge source controlled by the voltage U_z. It can be modeled with a SDD in Agilent-ADS.

III. STATIC BEHAVIOR

A simulation in the time domain has been done to check the membrane mechanical behavior. It is well known that the max displacement of the membrane switch is third of initial height, for an applied voltage just below the pull down voltage V_p and that the membrane becomes unstable for voltage higher or equal to V_p. The simulation has been done with a square voltage U_c, where U_{cmax} is just below V_p. All the parameters and their values used to make the simulations are given in Table II. With these parameters the computed pull down voltage V_p is 11.65 V. The simulation results are presented in Fig. 3 and Fig. 4. It is shown, in Fig. 3, that for an initial height of 0.9 µm the membrane displacement is 0.3 µm, when U_{cmax} is applied. The Fig. 4 shows that the membrane becomes unstable for an applied voltage above V_p.

TABLE II
MEMS SWITCH PARAMETERS

Membrane length	l	300 µm
Membrane width	w	35 µm
Membrane thickness	t	2.2 µm
Membrane initial height	h_0	0.9µm
Bottom electrode width	W	100 µm
Dielectric permittivity	ε_r	3
Dielectric thickness	t_d	0.1µm
Metal Poisson coefficient	ν	0.42
Metal density	ρ	19320 $kg.m^{-3}$
Metal Young modulus	E	$80E^9$ Pa
Air viscosity	µ	$1.8E^{-5}$ $kg.m^{-1}.s^{-1}$
Biaxial residual stress	σ	0 Pa

The mechanical behavior of the membrane computed with the model is just close to the real behavior showing that rise time and fall time of the bridge are not symmetrical. It is due to the difference between the electrostatic force pulling the membrane down and the restoring force pulling the membrane back up.

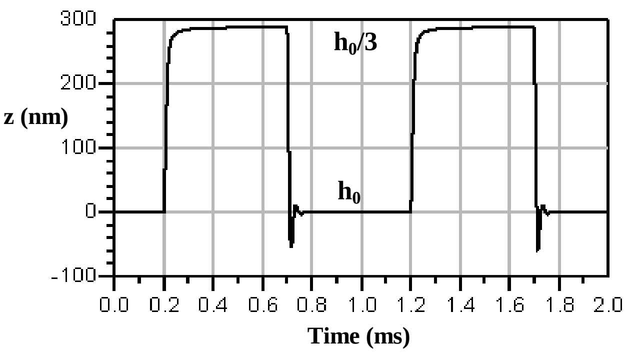

Fig. 3. Transient simulations of the membrane's behavior for a square voltage U_c with U_{cmax} just below V_p and U_{cmin} equal to 0. For z=0, the membrane is in the up state.

An S parameters simulation has been done to show the capacitance variations as a function of applied voltage U_c. The results Fig. 5 show that for a voltage going from 0 V to 11.6 V the capacitance goes from 33 fF to 47 fF.

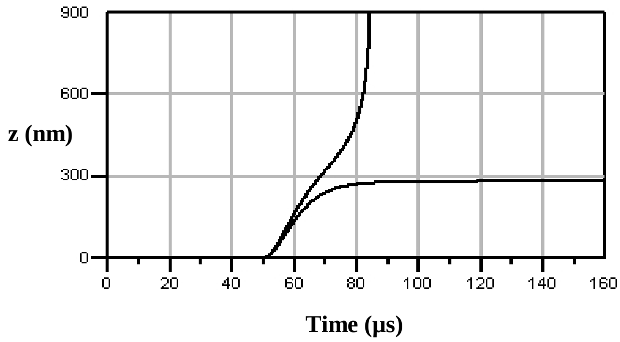

Fig. 4. Transient simulations of the membrane's behavior for a pulse voltage U_c just below V_p and just above V_p. For z=0, the membrane is in the up state.

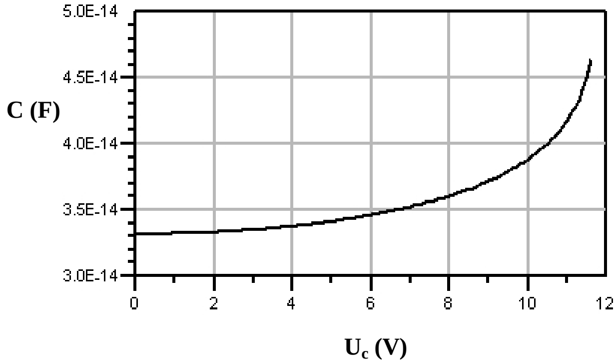

Fig. 5. Simulation of the capacitance of the switch C (F) as a function of applied voltage U_c(V).

III. DYNAMIC BEHAVIOR

A simulation in time domain with a low frequency source and a RF source can not be done because it would take too much time as it requires millions of points per period to obtain a result. But, as the model is constructed of voltage-controlled sources, a harmonic balance simulation, widely used in RF non-linear circuit simulation, allows to obtain easily the output spectrum of the circuit.

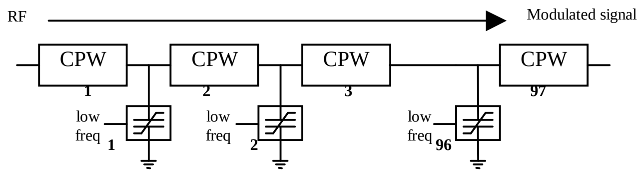

Fig. 6. Distributed MEMS transmission line BPSK modulator.

To show that the dynamic behavior can be studied, the model is included in a distributed MEMS transmission-line BPSK modulator as a single electrical element. The BPSK modulator of the Fig. 6, studied by Barker and Rebeiz[6], is designed with 96 MEMS capacitive switch models and with 97 lengths of CPW lines. The lines length is 197 µm, the substrate is 500 µm of quartz (ε_r=3.8), W=100 µm, G=100 µm. The switch parameters are given in Table II.

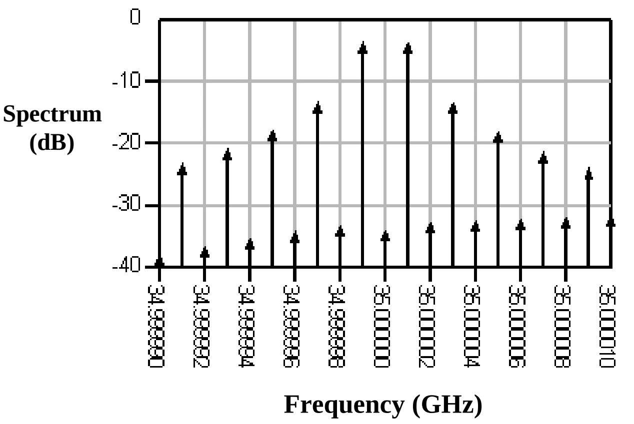

Fig. 7. Simulated spectrum of the distributed MEMS transmission-line BPSK modulator.

A S parameters simulation is done to obtain the low frequency amplitude voltage, which allows to have a phase shift of 180°. This simulation gives 11.6 V to obtain a phase shift of 180°. Barker and Rebeiz measured a pull down voltage V_P of 6.1 V and a modulating amplitude voltage of 6 V to obtain a phase shift of 180°.

The difference between their measurements and the model results is due to compressive stress on the beams[5]. The difference of the pull down voltage does not change anything in the capacitance value and it has no consequence on the RF simulation results. The harmonic balance simulation is then done with a 0-11.6 V 1-kHz square, as modulating signal, and RF at 35 GHz.

The spectrum of the modulator is presented Fig. 7. The fundamental frequency is lower than the first harmonic, and the harmonics are not symmetrical although the difference between the amplitude harmonic is small. The harmonics must not be symmetrical because the rise time and the fall time of the membrane are different. But the difference is small because the square modulating signal is approximated and depends strongly on the number of harmonics used in the simulation. As can be seen in Fig. 8 the approximated modulated signal causes amplitude oscillations of the membrane displacement.

These oscillations are causing modifications of the rise and fall time, which are not very different one to the other. The spectrum with the harmonic amplitude is presented Table III. This can be improved by using more harmonics.

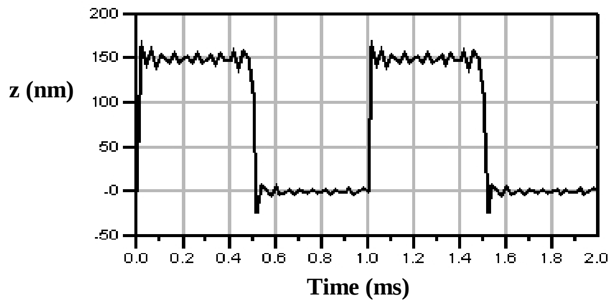

Fig. 8. Behavior of the membrane, when a modulating signal approximated with 30 harmonics is applied.

IV. Conclusion

Electromechanical model of MEMS switches has been demonstrated with non-linear electrical equivalent sources. It allows to predict the static and the dynamic behavior of the membrane and to perform S parameters, transient or harmonic balance simulations, using a single tool. The model has been included in a BPSK modulator leading to good results.

TABLE III
RF Spectrum of the BPSK Modulator

f_0	f_{-1}	f_{-3}	f_{-5}	f_{-7}	f_{-9}
-34.1	-3.7	-13.3	-17.9	-20.9	-23.3
f_0	f_1	f_3	f_5	f_7	f_9
-34.1	-3.8	-13.6	-18.3	-21.5	-24.1

References

[1] J.Y. Qian, G.P. Li, F De Flaviis, "A parametric model of MEMS capacitive switch operating at microwave frequencies", *IEEE MTT-S int. Microwave Symp. Dig.*, Boston, MA, pp. 1229-1232, June 2000.
[2] L. Vietzorreck, F. Coccetti, V. Chtchekatourov and P. Russer, "Modeling of MEMS capacitive switches by TLM", *IEEE MTT-S int. Microwave Symp. Dig*., Boston, MA, pp. 1221-1224, June 2000.
[3] T. Veijola, "Large-displacement modeling and simulation of micro-electromechanical electro-statically driven resonators using the harmonic balance method", *IEEE MTT-S int. Microwave Symp. Dig.*, Boston, MA, pp. 99-102, June 2000.
[4] E.K. Chan, E.C Kan and R.W. Dutton, "Nonlinear dynamic modeling of micromachined microwave switches", *IEEE MTT-S int. Microwave Symp. Dig.*, Denver, CO, pp. 1511-1514, June 1997.
[5] N.S. Barker, "Distributed MEMS transmission lines", Ph.D. dissertation, Univ. Michigan, Ann Arbor, 1999.
[6] N.S. Barker and G.M. Rebeiz, "Distributed MEMS transmission-line BPSK modulator" *IEEE Microwave and Guided Wave Letters*, vol. 10, no. 5, May 2000.